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ciples I have been advocating, be generally very different from that usually stated as such at the present day. Nevertheless, I would submit that the principles are sound, and should be as far as is practicable adhered to. They demand no more than that the value in question should depend, amongst other considerations, upon the constitution of the Company with which the contract is made, its progress or condition in the interval between the commencement and termination of such contract, and the degree of abruptness with which this last is brought to a conclusion,—elements which, it can scarcely with justice be denied, should enter into the determination of such a question.

On the Contrivances required to render Contingent Reversionary Interests Marketable Securities.

[Continued from No. 3.]

THE third question proposed to Mr. Sang is as follows:—

B, aged 30, requires an advance of £100; and in consideration thereof, offers to pay to the lender an annuity during his life after A's death. Required the annuity to be demanded, and the sum to be assured.

The point here proposed for consideration is not the mere amount of the annuity to be charged, the computation for which is sufficiently simple, but the nature and amount of the assurance.

An annuity, to be paid at stated terms during B's survivorship of A, is now worth £4·981 for each £1 of annuity, computations being made at 6 per cent. Hence, for a present advance of £100, an annuity of £20·076 would need to be secured out of the rents, supposing that no reassurance is required.

It is by no means an easy matter to discover any principle which ought absolutely to determine the nature and amount of the assurance. If we suppose that the principle which is to be followed out is that of converting a contingent into an absolute reversion, we may reason as below.

Let C be the residuary legatee of a large property burdened with an annuity during the life of A, and thereafter during that of B. The present value of the two annuities forms a burden on the property, which burden is gradually removed until it be altogether extinguished. The extinction of that burden may thus be regarded as an absolute reversion, of which the contingencies possessed by A and by B form two parts. The death of one of the annuitants will relieve the estate of a part of the burden, and the death of the survivor will remove the burden entirely; and the amount of this relief is the complementary reversion which must be met by reassurance. Any assurance, therefore, conducted on this principle, must be too complex in its arrangements to be brought into actual use. The sum to be assured at B's death is thus left indeterminate, so far as any strict principle is concerned, and must be fixed arbitrarily by some conventional rule. All therefore that can be stated on this head is, that after the amount of reassurance is determined on, the loss caused by it must be computed as has been already shown in the first part of this paper; and that it appears reasonable to secure by assurance that sum which a present advance would purchase at B's succession to A.

The fourth question is as follows:—

B, aged 30, requires an advance of £100; for which he proposes to grant an obligation to pay an annuity during his survivance of A, but with the option of redeeming that annuity at A's death,—the annuity and redemption-money to be both fixed now. What sum and what annuity should be demanded? Also, what sum should be secured from the commencement to cover such risk throughout? As, with such an option, it is presumed that assurance must be opened on the whole life scale, the actuary will have the goodness to state, whether he considers the borrower, in the event of his redeeming at the fixed sum, entitled to have the policy transferred to him.

It is here proposed to allow B to choose, at his succession to A, whether he shall pay down a certain sum, or shall continue to pay an annuity during his life,—the amount of the redemption-money and the amount of the annuity to be fixed now. We must assume that, in making his selection, B will take that course which shall then appear most advantageous to himself. If his succession occur early, he will prefer to pay down the fixed sum, as he will then be able with advantage to borrow money for that purpose; while, if the succession occur when he is advanced in life, he will then choose rather to pay the annuity.

There is a certain age at which it will be to him a matter of indifference; and therefore the lender of the money may proceed as if there were no option, and as if it were agreed that before this age a single payment is to be made, but that after this age the payment is to be made by annuity.

Here we must not omit one element of the consideration, which is yet too little known to be allowed for; viz., the chance of B being in bad health at the time of exercising the option. This element is one of considerable weight; but there are no data known from which the value of it may be deduced.

The ratio which the annuity bears to the single payment must determine the *neutral* age, as we may call it. So that the problem may be solved in many different ways, according to the ratio which may be adopted, or to the neutral age which may be selected.

The choice of either of those is quite arbitrary, and the problem is thus a pure porism; yet it is possible that the following considerations may guide those actually engaged in such transactions.

In the present instance, the sum which must be secured at B's succession to A, is £247·53 for £100 of present advance; while the annuity to be assigned for the same advance is 20·076. The redemption-money is thus 12·329 years' purchase of the annuity; so that if the above sums, or sums in the same ratio, were now to be fixed, the neutral age, or the age at which the selection would be a matter of indifference, would be 41.*

Whether this consideration be or be not sufficient to determine the selection of the ratio 12·329 (or of the neutral age 41), the adoption of these will not diminish the entire generality of the succeeding steps of the investigation, which steps are equally applicable to any other arbitrary assumption.

The question now becomes this, to compute what assurance at A's death, if it happen before B's within 11 years, together with an annuity during B's survivorship, if A's death happen after 11 years, may have the present value of £100,—the assurance being 12·329 times the annual payment.

The present value of £1, to be paid at B's succession to A, if it happen within 11 years, is £·23765; so that the present value of the above-named sum of £247·53, secured in the same way, must be £55·823.

An annuity during B's survivance of A, provided his succession take place after 11 years, is not simply a deferred survivorship annuity; its present value must be sought by computing the value as at ages 41 and 69 of an annuity during the survivorship of B, and then discounting that for 11 years, allowing also for the probability of the group 30—58 existing at the end of that time. Hence the following computation:—

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* It must here be observed, that the survivorship annuity must be taken as having its first payment six months after the date of succession.

Annuity at Age 41 = 12·88970
Joint Annuity, 41 and 69 = 6·76678

Survivorship of 41 . = 6·12292 . Log. = 0·78696
Log. Joint Endowment, 11 years hence . „ = 9·48729
Log. £20·076 . . . . „ = 1·30268

Log. £37·751 . . . . „ = 1·57693

Whence the value of the second part of the expectation comes out £37·751, which, added to the preceding £55·823, makes in all £93·574. The option thus reduces the present value from £100 to £93·574, or by rather less than 6½ per cent.; and in order to bring the present value up again to £100, we must augment both the redemption-money and the annuity in the ratio of 100 : 93·574. Hence

Log. 247·53 = . . . 2·39361
Colog. 93·574 = . . . 0·02884
Log. 20·076 = . . . 1·30268

Log. £264·52 = . . . 2·42245
Log. £21·455 = . . . 1·33152

That is, for the present advance of £100, and also for the advantage of having an option, B must engage to pay, at his succession to A, the sum of £264·52, or else an annuity during his life thereafter of £21·455,—the option being exercised immediately after A's death.

The subject of reassurance in such a transaction falls next to be considered. In the absence of any complete guide in this matter, we may presume that an assurance of the fixed redemption-money, payable at the death of B, is sufficient to induce the lender to advance the money. Returning then to the amount which would need to be secured were there no option, viz., £247·53, we must ascertain the loss which would be occasioned by such an assurance. The 3 per cent. premium for such an assurance is £4·9035, which, multiplied by the value of a 6 per cent. annuity, gives £68·748; while the value (at 6 per cent.) of the assurance is £52·606. Hence there is a loss of £6·142 by the assurance, which loss falls to be deducted from the £93·574, leaving £87·432. The joint effect of the option and of the assurance is thus to reduce the value in the ratio of £87·432 to £100. The amount must therefore be increased in the same ratio. Hence

Colog. ·87432 = . . . 0·05833

Log. 247·53 = . . . 2·39361
Log. 20·076 = . . . 1·30268
Log. 4·9035 = . . . 0·69051

Log. 283·10 = . . . 2·45194
Log. 22·962 = . . . 1·36101
Log. 5·6084 = . . . 0·74884

Whence it follows that B must engage to pay £283·10 at his succession to A, or else an annuity of £22·962 at stated terms thereafter; while the lender pays an annual premium of £5·6084 for the policy of assurance.

This policy of assurance, even though B should pay the £283·10, most clearly belongs to the lender, and cannot be claimed by B, unless a fair equivalent be paid for its transference. The lender has indeed paid for it all along; and it is manifestly unjust that he should then lose the benefit of it. As however it is doubtful that, in the present state of the law, the lender could recover the sum at B's death, from the Assurance Office, it is proper that the proceedings in regard to the policy be clearly laid down in the original contract.

If, in that contract, it be agreed that the policy of assurance is to be transferred to the borrower, in the event of his redeeming, the whole form of the computation becomes changed, since the considerations which will determine the selections are entirely different. B will then consider whether it be to his advantage to pay the redemption-money, less the value of the policy to which he would then be entitled, or to pay the annuity. In this case the problem becomes doubly porismatic, since the sum to be assured is also arbitrary. The ratio of the redemption-money to the annuity, and the ratio of the same redemption-money to the sum assured, having been both settled arbitrarily, or by such considerations as may present themselves to the parties, the computations may be proceeded with. But these would be so excessively complex as to be almost out of the question; and the writer, not anticipating the occurrence of such a case in practice, has not proceeded with the details.

The following is the fifth question :—

B wishes to have the option of redeeming at any time after the death of A. What arrangement should be made to allow the redemption-money to remain the same as originally fixed, in the event of the annuity being redeemed, after it has been paid for a number of years? It is presumed that such an arrangement could be equitably made by a contract to hand over the whole life policy, the increasing value of which would act as a modification of the terms of redemption from year to year.

The problem here proposed differs from the preceding in this, that it allows the borrower to exercise the selection at any time after the death of A; and thus a new consideration is introduced, so that the problem appears more porismatic than ever. But this is again modified by the new consideration, that the right of selection is never at any time to have a value; the value of the annuity, plus that of the policy, being equal at all times to the fixed redemption-money.

From this last-mentioned restriction it can be shown, by a demonstration somewhat intricate, that there is only one possible solution; and that solution has already been anticipated in Case 6 of the above questions. It is therefore unnecessary to say more on the present head.

The last question proposed is as follows:—

On what terms in any of the foregoing cases should the parties be allowed to cancel the whole transaction, A and B being both alive at the time?

This, theoretically, presents little difficulty, although some apparent inconvenience may arise in practice. We have only to compute the values of the obligations on each side; and the difference of these values will then show what must be repaid in order that the whole transaction may be cancelled.

There is nothing in the mode of conducting the calculations which calls for particular notice: but the result of the computation is, in general, such that the lender would have but a very small return for his advances; and thus the Offices are induced to add considerably to the computed price of surrender.

It ought however to be kept in mind, that the lender has enjoyed the chance of making a considerable gain, and that this must be taken along with the price of surrender as forming the entire return for his money.

It must also be remarked, that if the borrower have stipulated for a right of redemption before the death of A, according to a computed price, he has a very great advantage in judging of the course which he may pursue, according to the state of A's health.

The Office has no direct means of ascertaining this; neither can A be required to answer truly questions put to him,—nor indeed could false answers by him be laid to the charge of the borrower. A stipulation that A must be in his usual health at the time of surrender, will thus only partially counteract the force of selection by B; and therefore the lender ought to be very cautious in granting power of redemption to B, while A may be alive.

In strictness, the computations for such a surrender ought to be based upon tables showing the benefit of selection. There are, however, no such tables in existence;* and the writer is again under the necessity of regretting, that the attention of those engaged in Life Offices has never been turned to this very important and interesting subject.†

Sketch of the recent Progress of the Assurance of Life and Property on the Continent. By SAMUEL BROWN, Esq., *Member of the Council and one of the Honorary Secretaries of the Institute of Actuaries, and Actuary of the Mutual Life Assurance Society.*

[Read before the Institute of Actuaries, 30th June, 1851.]

PART I.—FRANCE.

IN laying before the members of the Institute of Actuaries this hasty sketch of the Progress of the Assurance of Life and Property on the Continent, I beg to offer a few words of explanation on what induced me to attempt it.

Understanding that several actuaries and managers of Foreign Assurance Companies were likely to be in England at this time, without any general acquaintance with, or means of introduction to, those who are engaged in similar pursuits and studies in this country, the council of the Institute were led to propose a general meeting and conference, and to offer to them the hospitality and

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* This desideratum has been to a certain extent recently supplied by Mr. Edwin James Farren in his work on Life-Contingency Tables, and by Mr. Higham in two papers read by him before the Institute of Actuaries.—ED. *A. M.*

† We have given the whole of Mr. Sang's observations on this subject, rather for the sake of the suggestive character of the discussions in them, than for their practical utility. In a future number we hope to show methods by which the requirements of transactions such as those here referred to may be met with great simplicity and exactness.—ED. *A. M.*